

Appendix A

The primitive \mathcal{F}

$$\begin{aligned}
\mathcal{F} = & \frac{1}{1209600.0} \{ 1323 \cos(5\sqrt{k}L) - 675 \cos(7\sqrt{k}L) \\
& + \sqrt{k}L(378000 \sin(\sqrt{k}L) + 21000 \sin(3\sqrt{k}L) - 7560 \sin(5\sqrt{k}L)) \\
& + \sqrt{k}l^*[23625 \sin(\sqrt{k}L) + 4725 \sin(3\sqrt{k}L) - 14175 \sin(5\sqrt{k}L) + 4725 \sin(7\sqrt{k}L) \\
& + \sqrt{k}L(-37800 \cos(5\sqrt{k}L)) \\
& + (\sqrt{k}L)^2(-75600 \sin(3\sqrt{k}L) + 226800 \sin(\sqrt{k}L))] \\
& + (\sqrt{k}l^*)^2[-49707 \cos(5\sqrt{k}L) + 14175 \cos(7\sqrt{k}L) \\
& + \sqrt{k}L(1587600 \sin(\sqrt{k}L) - 172200 \sin(3\sqrt{k}L) + 68040 \sin(5\sqrt{k}L))] \\
& + (\sqrt{k}l^*)^3[-80325 \sin(\sqrt{k}L) - 144725 \sin(3\sqrt{k}L) + 82215 \sin(5\sqrt{k}L) - 23625 \sin(7\sqrt{k}L) \\
& + \sqrt{k}L(37800 \cos(5\sqrt{k}L)) \\
& + (\sqrt{k}L)^2(680400 \sin(\sqrt{k}L) - 126000 \sin(3\sqrt{k}L))] \\
& + (\sqrt{k}l^*)^4[68985 \cos(5\sqrt{k}L) - 23625 \cos(\sqrt{k}L) \\
& + \sqrt{k}L(2041200 \sin(\sqrt{k}L) - 205800 \sin(3\sqrt{k}L) + 37800 \sin(5\sqrt{k}L))] \\
& + (\sqrt{k}l^*)^5[-458325 \sin(\sqrt{k}L) - 43225 \sin(3\sqrt{k}L) - 25893 \sin(5\sqrt{k}L) + 14175 \sin(7\sqrt{k}L) \\
& + \sqrt{k}L(68040 \cos(5\sqrt{k}L)) \\
& + (\sqrt{k}L)^2(680400 \sin(\sqrt{k}L) - 25200 \sin(3\sqrt{k}L))] \\
& + (\sqrt{k}l^*)^6[-945 \cos(5\sqrt{k}L) + 4725 \cos(7\sqrt{k}L) \\
& + \sqrt{k}L(831600 \sin(\sqrt{k}L) - 12600 \sin(3\sqrt{k}L) - 37800 \sin(5\sqrt{k}L))] \\
& + (\sqrt{k}l^*)^7[(-354375) \sin(\sqrt{k}L) + 5425 \sin(3\sqrt{k}L) - 1323 \sin(5\sqrt{k}L) - 675 \sin(7\sqrt{k}L) \\
& + \sqrt{k}L(-7560 \cos(5\sqrt{k}L)) \\
& + (\sqrt{k}L)^2(226800 \sin(\sqrt{k}L) + 25200 \sin(3\sqrt{k}L))] \\
& + \cos(\sqrt{k}L)((\sqrt{k}l^*)^2 + 1)(4725)[(\sqrt{k}l^*)^5(\sqrt{k}L)(80) \\
& + (\sqrt{k}l^*)^4(155 - 48(\sqrt{k}L)^2) \\
& + (\sqrt{k}l^*)^3(\sqrt{k}L)(64) \\
& + (\sqrt{k}l^*)^2(182 - 96(\sqrt{k}L)^2) \\
& + \sqrt{k}l^*(-16\sqrt{k}L) \\
& + (\sqrt{k}L)^2(-48) + 75] \\
& + \cos(3\sqrt{k}L)(-175)[(\sqrt{k}l^*)^7(\sqrt{k}L)(120) \\
& + (\sqrt{k}l^*)^6(3)(144(\sqrt{k}L)^2 + 71) \\
& + (\sqrt{k}l^*)^5(\sqrt{k}L)(744) \\
& + (\sqrt{k}l^*)^4(720(\sqrt{k}L)^2 + 347) \\
& + (\sqrt{k}l^*)^3(\sqrt{k}L)(-24) \\
& + (\sqrt{k}l^*)^2(144(\sqrt{k}L)^2 - 473) \\
& + \sqrt{k}l^*(-648)\sqrt{k}L \\
& + (\sqrt{k}L)^2(-144) - 31]\}
\end{aligned}$$

In order to confirm that the code implementation gave the same result than the original double integral, random values were assigned to $\sqrt{k}l^*$ and $\sqrt{k}L$ and both expressions, the solved and the double integral, were numerically evaluated with difference lower than 10^{-3} relative.